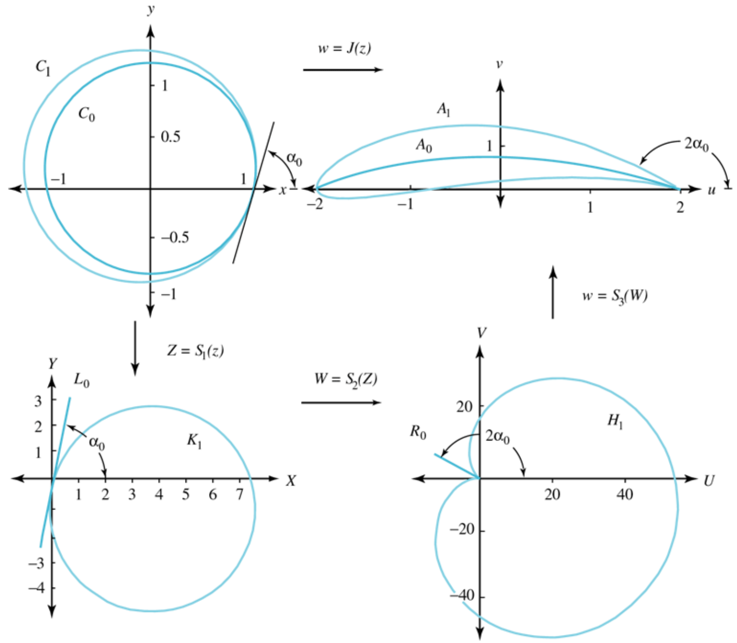
GEOMETRIE

Simulation von Strömungen

AUFGABEN UND ZIELE

- Wie stelle ich Strömungen dar?
- Welche Elementarströmungen gibt es?
- Wie erhalte ich ein Airfoil Profil?
- Joukowsky Transformation



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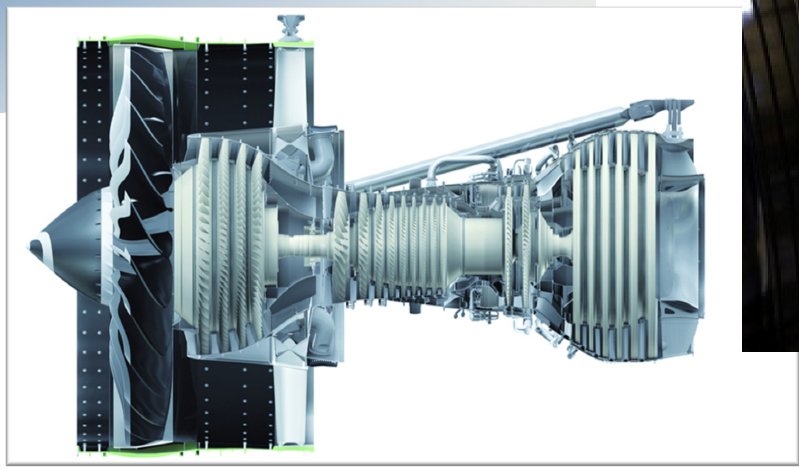
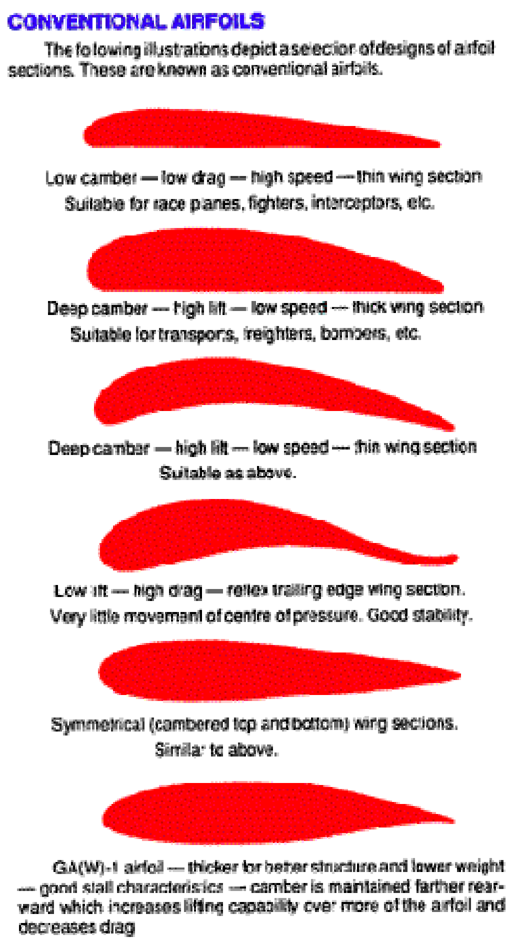
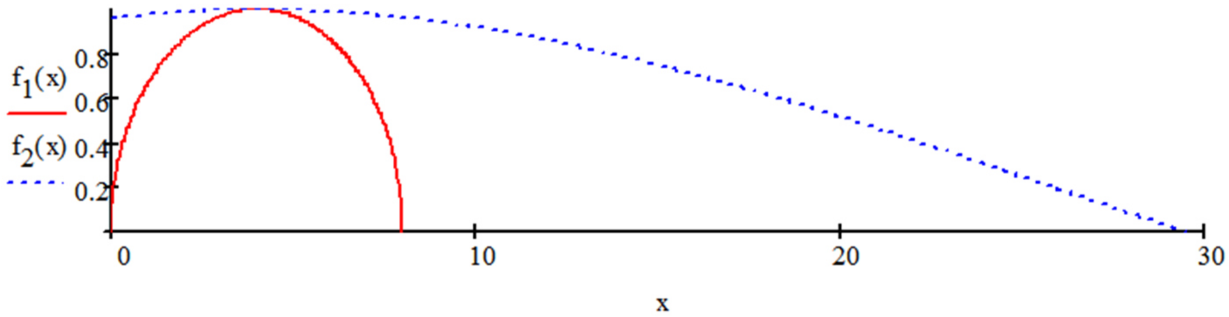
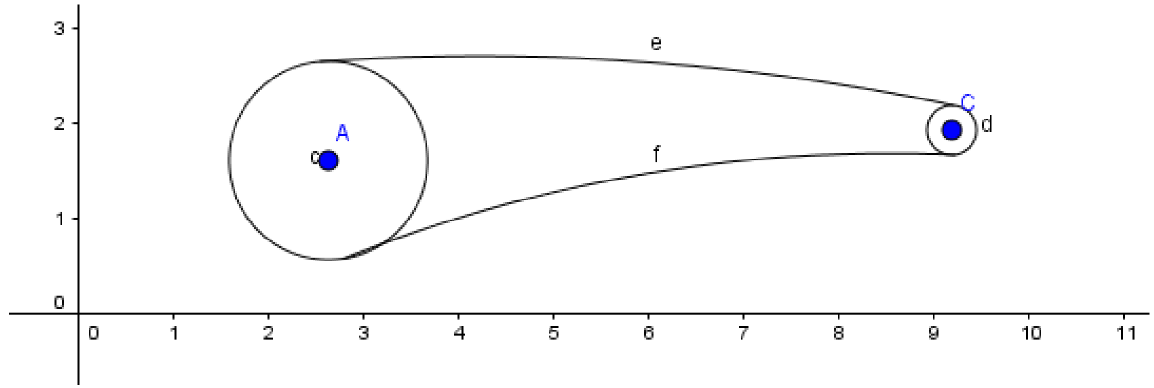
AIRFOIL



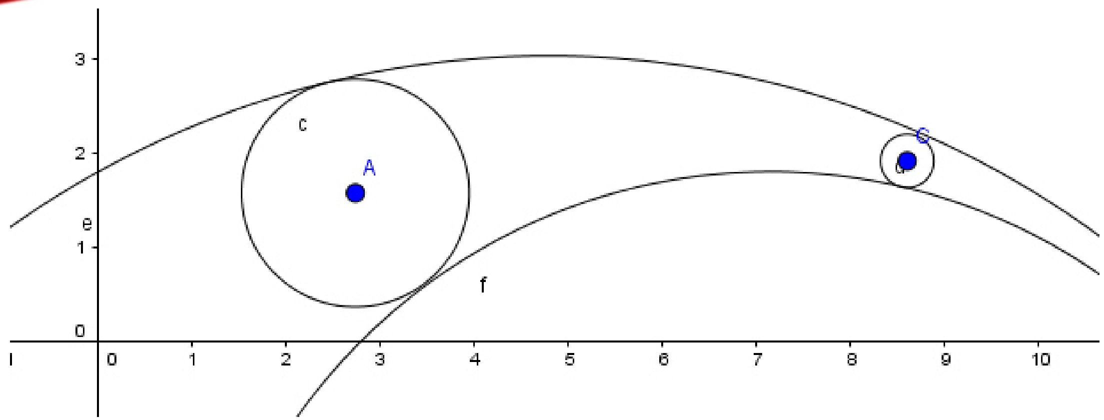




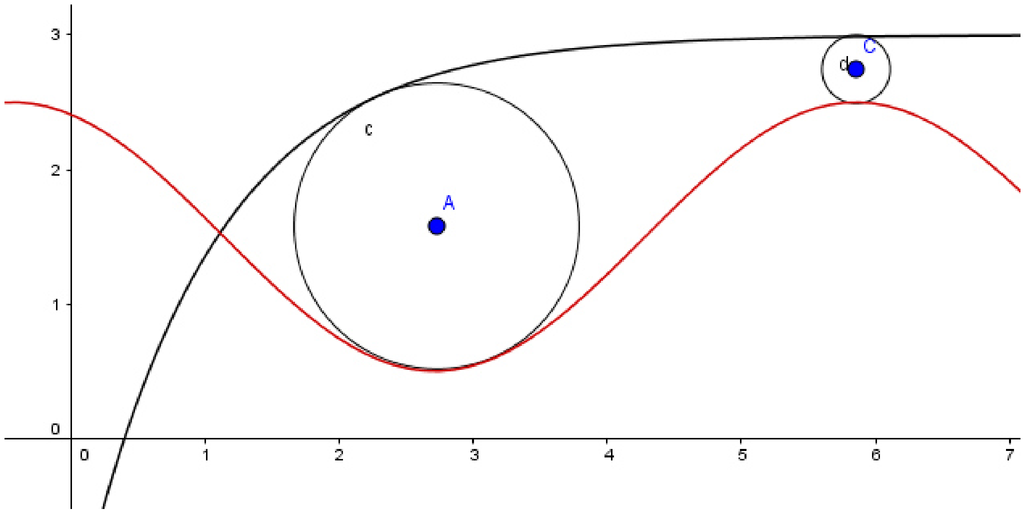
Ellipse + Polynomfunktion (Grad 2)



Kreise & Polynominterpolation (Grad 2 & 4)



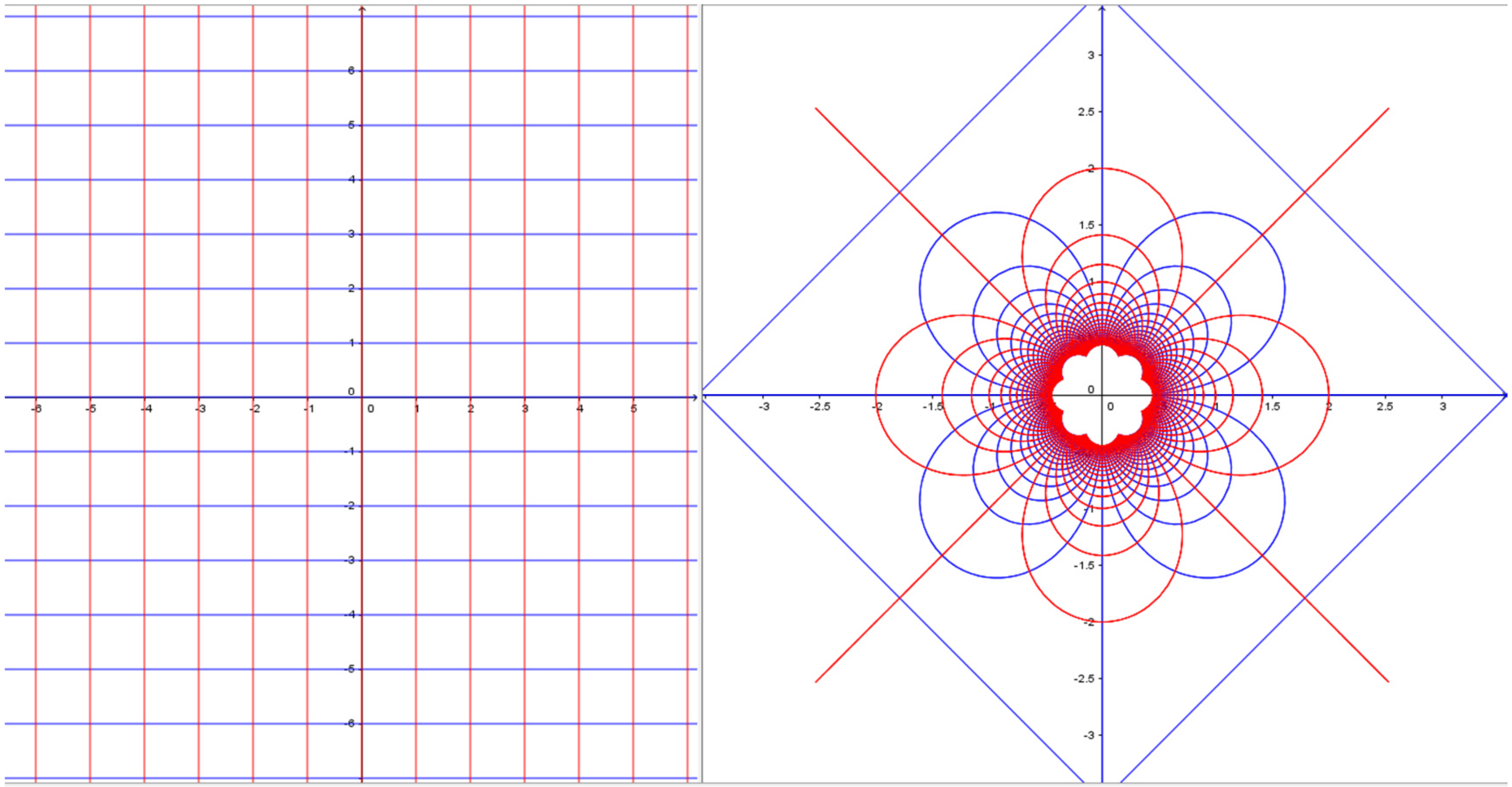
Zusammensetzten von
Kreisen bzw. Kreisbögen



Kreise + Log-Funktion
+ Sinus-Funktion

FUNKTIONEN MIT KOMPLEXEN ZAHLEN

$F(c)=\sqrt{\frac{2}{c}}$



JOUKOWSKY TRANSFORMATION EINES KREISES

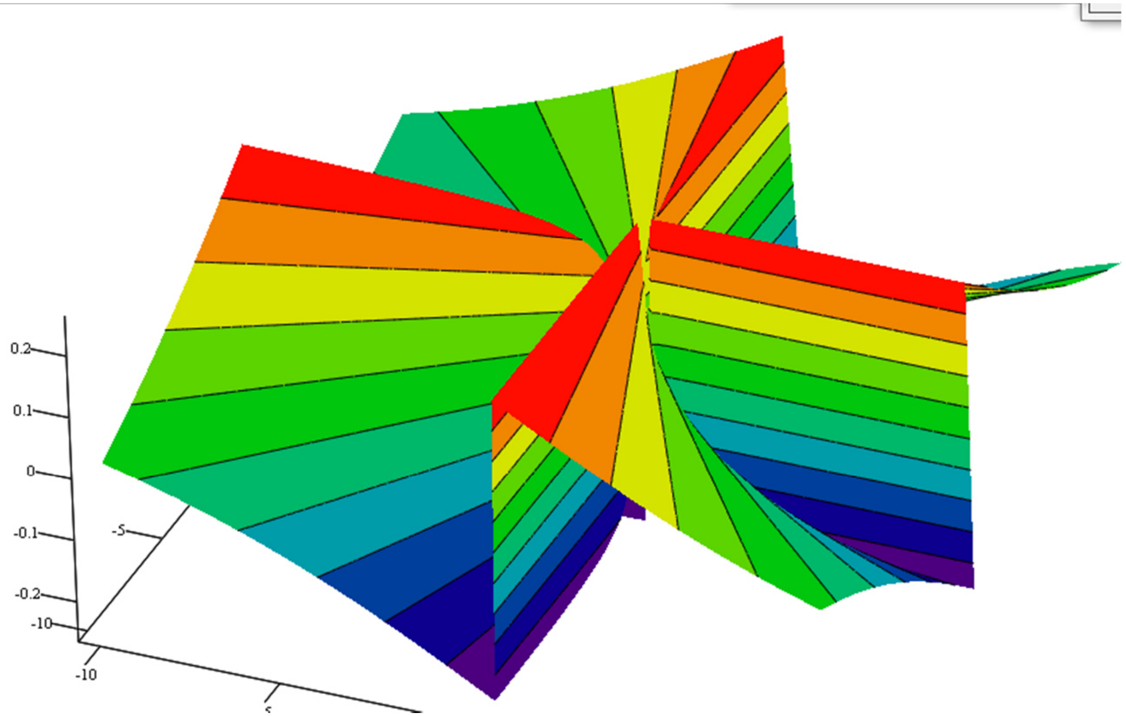
- Funktion zum Umwandeln von Kreis zu Airfoilprofil
- Joukowsky Transformation: $f(z) := z + \frac{1}{z}$ $\quad f(z) := x + \frac{x}{x^2 + y^2} + i \cdot \left(y - \frac{y}{x^2 + y^2} \right)$
- In Polarkoordinaten:

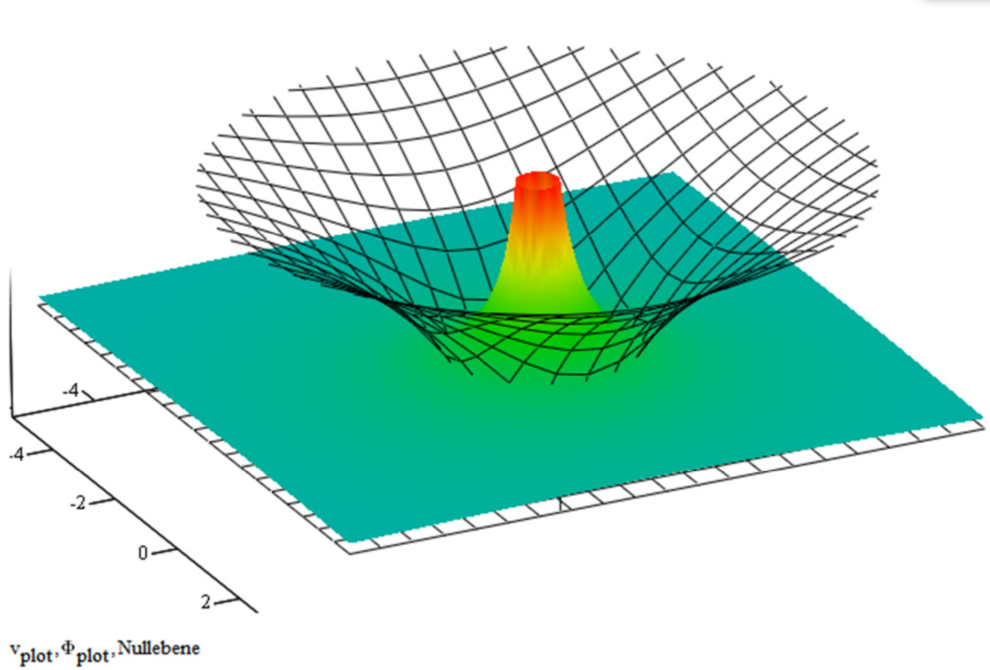
$$f(z) := r \cdot \cos(\phi) + \frac{r \cdot \cos(\phi)}{(r \cdot \cos(\phi))^2 + (r \cdot \sin(\phi))^2} + i \cdot \left(r \cdot \sin(\phi) - \frac{r \cdot \sin(\phi)}{(r \cdot \cos(\phi))^2 + (r \cdot \sin(\phi))^2} \right)$$

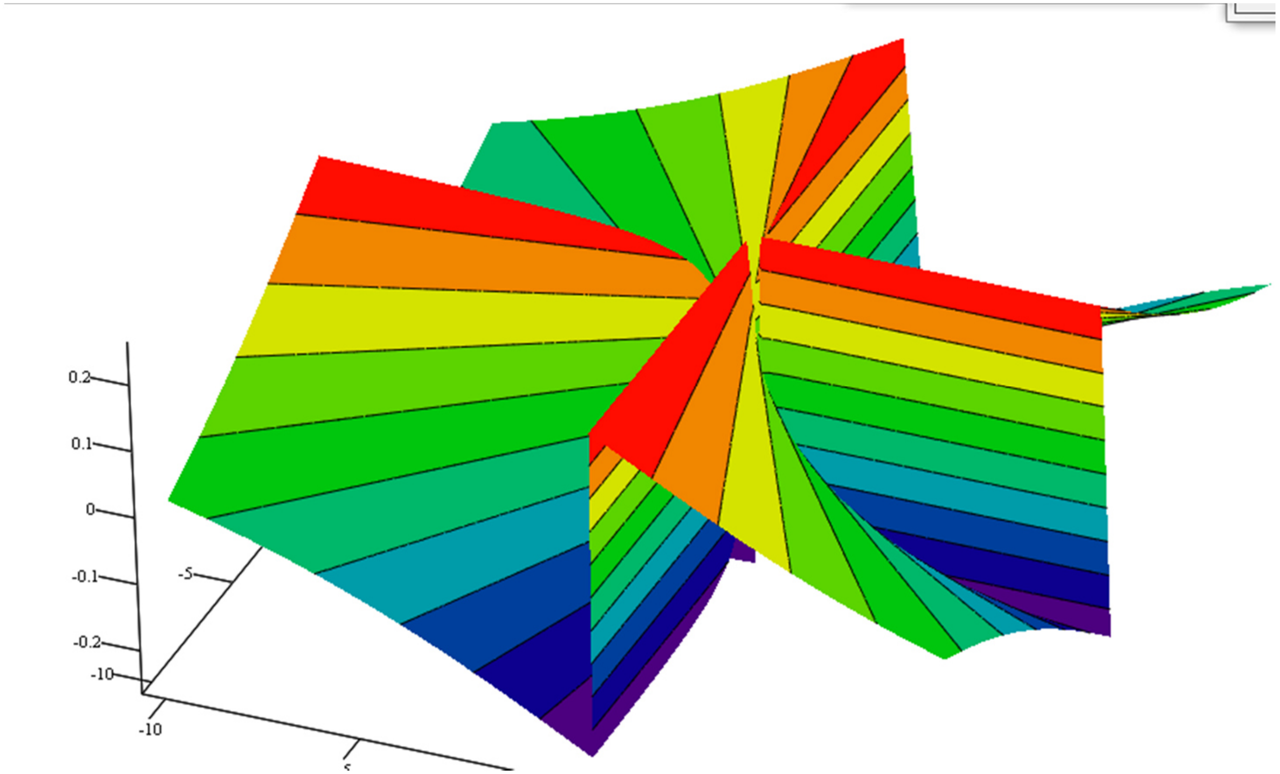
- Geogebra:
    - X-Achse: Realteil
    - Y-Achse: Imaginärteil

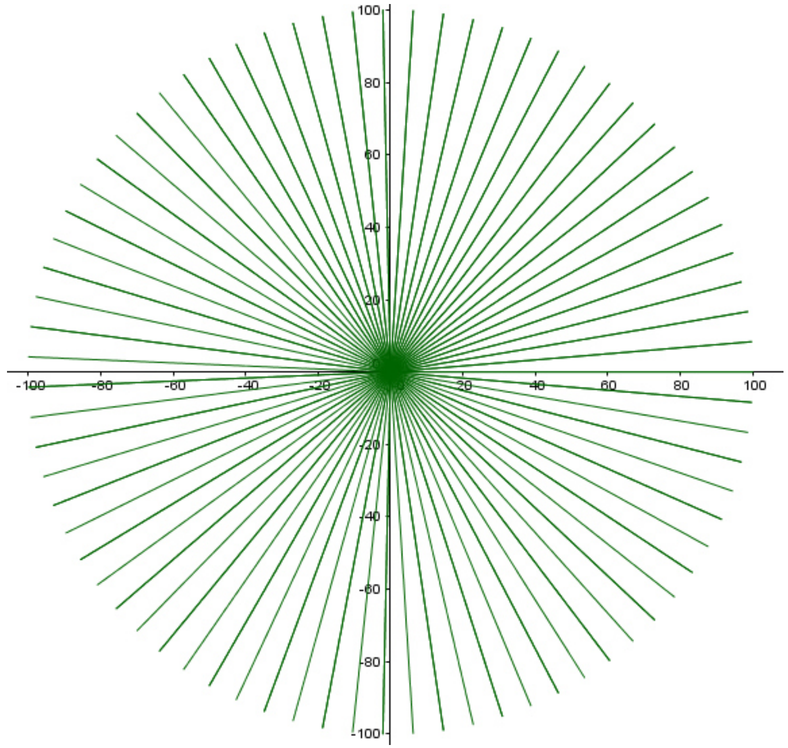
- Airfoil als Ergebnis

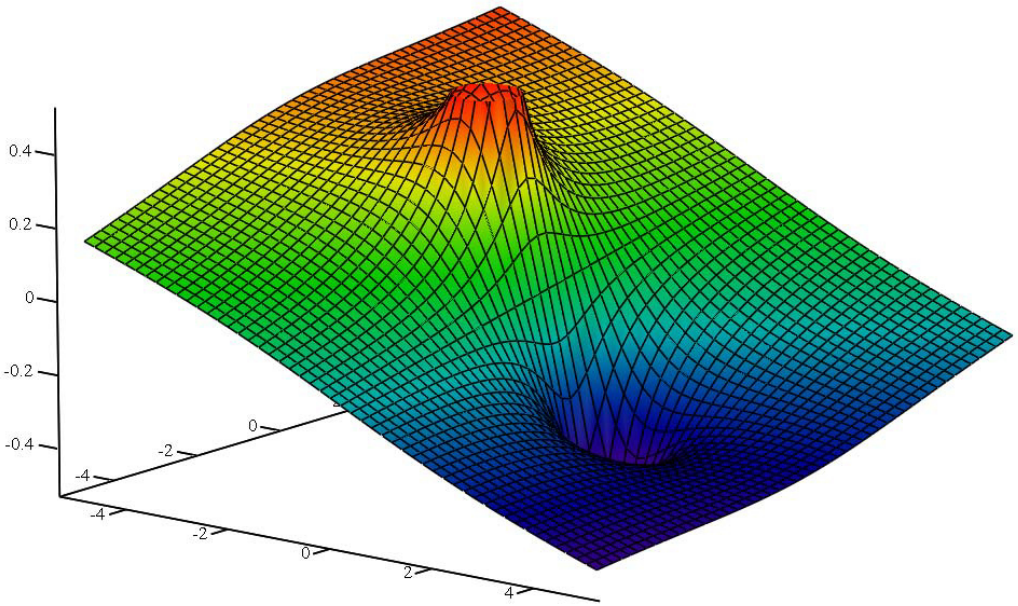
ELEMENTARSTRÖMUNGEN

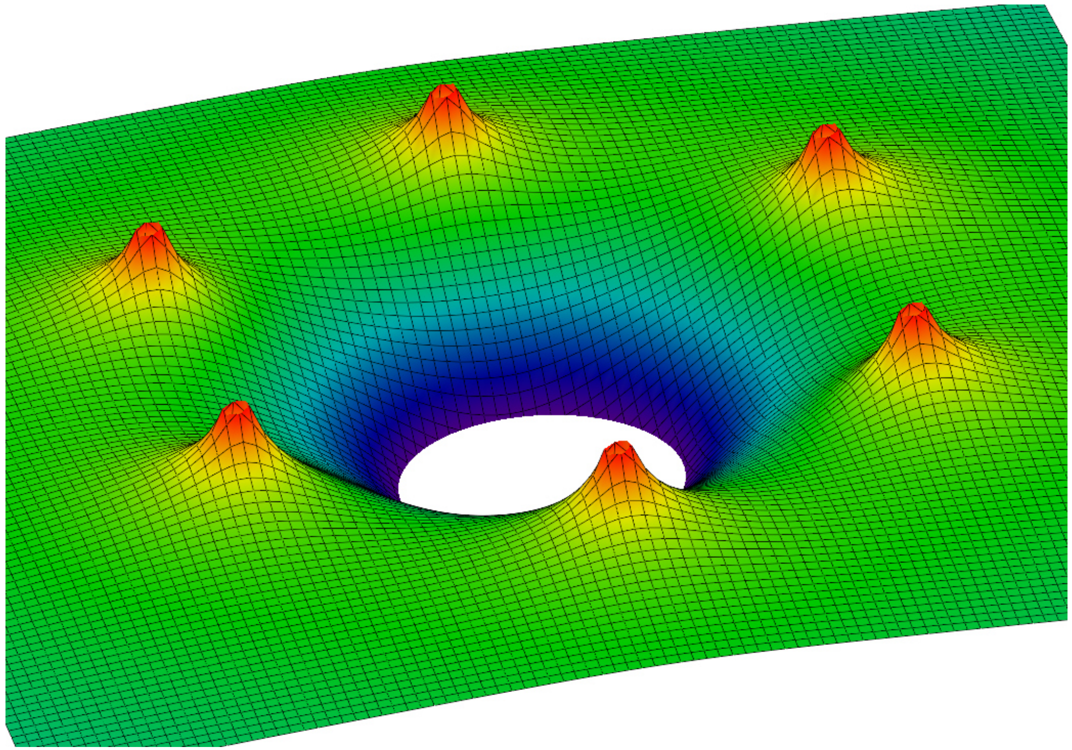





0.2
0.1
0
-0.1
-0.2
-10
-5
-10


100
80
60
40
20
-100
-80
-60
-40
-20
20
40
60
80
100
-20
-40
-60
-80
-100


0.4
0.2
0
-0.2
-0.4
-4
-2
0
-4
-2
0
2
4



SIMULATION VON STRÖMUNGEN AM AIRFOIL

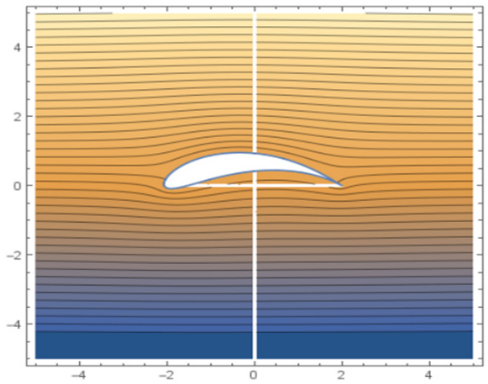
- Geogebra → nicht möglich
- Komplexere Programme → z. B. Wolfram Mathematica

```
In[23]:= Solve[w == z + 1 / z, z]
```

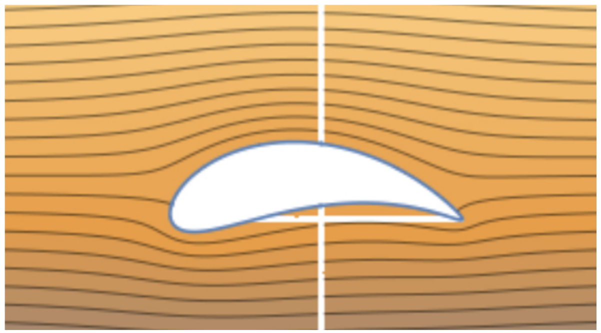
Out[23]= $\left\{\left\{z \to \frac{1}{2}\left(w - \sqrt{-4 + w^2}\right)\right\}, \left\{z \to \frac{1}{2}\left(w + \sqrt{-4 + w^2}\right)\right\}\right\}$

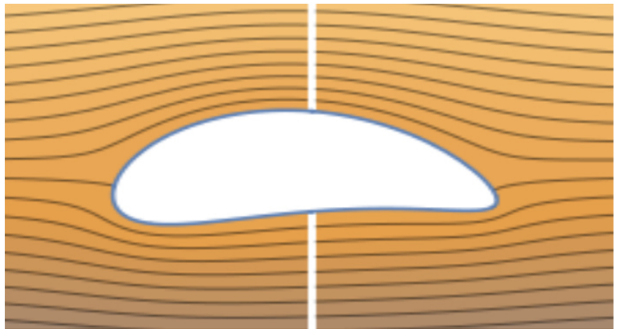
```
In[104]:= z1[a_, b_] := 1 / 2 * (a + ⅈ * b - Sqrt[(a + ⅈ * b) ^ 2 - 4]);
          z2[a_, b_] := 1 / 2 * (a + ⅈ * b + Sqrt[(a + ⅈ * b) ^ 2 - 4]);
          mp = -0.14 + 0.38 * ⅈ; R = 1.2; Uinf = 1.1;

In[107]:= p1 = ContourPlot[Uinf * (Im[z1[a, b]] - Im[mp]) * (1 - R ^ 2 / ((Re[z1[a, b]] - Re[mp]) ^ 2 + (Im[z1[a, b]] - Im[mp]) ^ 2)), {a, -5, 5}, {b, -5, 5}, Contours → Table[c, {c, -5, 5, 0.25}],
            RegionFunction → Function[{a, b, f}, Abs[z1[a, b] - mp] ≥ R]];

In[108]:= p2 = ContourPlot[Uinf * (Im[z2[a, b]] - Im[mp])  (1 - R ^ 2 / ((Re[z2[a, b]] - Re[mp]) ^ 2 + (Im[z2[a, b]] - Im[mp]) ^ 2)), {a, -5, 5}, {b, -5, 5}, Contours → Table[c, {c, -5, 5, 0.25}],
            RegionFunction → Function[{a, b, f}, Abs[z2[a, b] - mp] ≥ R]];

In[109]:= J = ParametricPlot[{Re[R * Exp[I * phi] + mp + 1 / (R * Exp[I * phi] + mp)], Im[R * Exp[I * phi] + mp + 1 / (R * Exp[I * phi] + mp)]}, {phi, -Pi, Pi}];

In[110]:= Show[p1, p2, J]
```

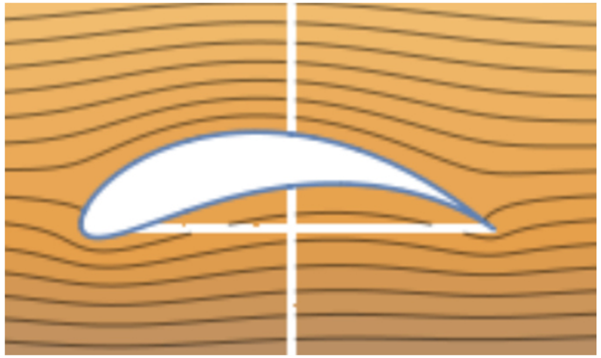
Out[110]=

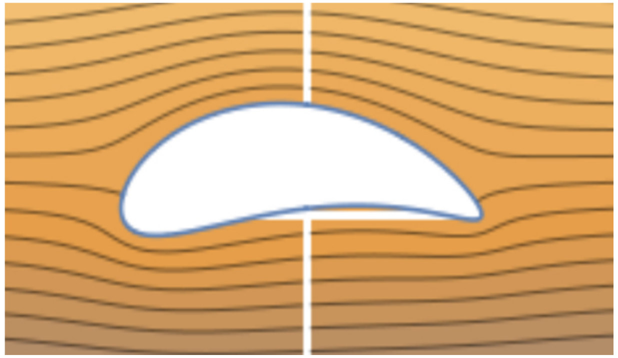


STRÖMUNGEN DURCH VERSCHIEDENE AIRFOILS



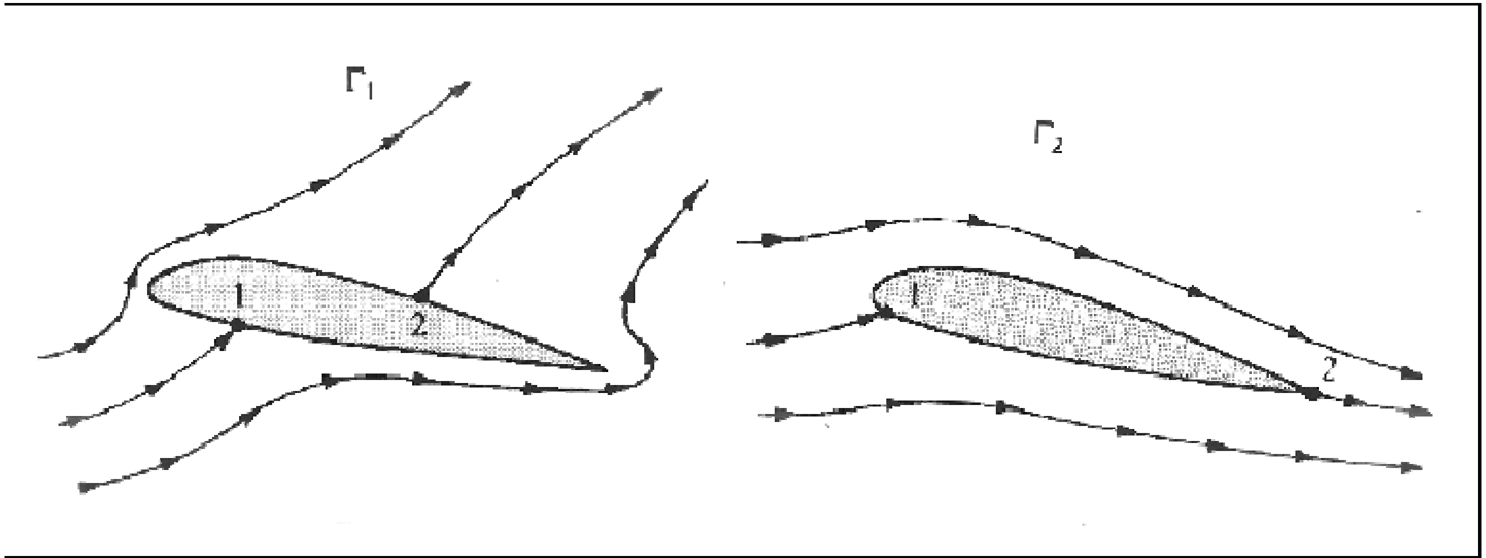






BETRACHTUNG VON GENAUEREN MODELLEN

- Kutta Bedingung:
  - Airfoil + Wirbelströmung



FÜR DIESE TOLLE WOCHE!