

Numerische Mathematik

Mathematik und Umwelt: Schadstoffsimulationen

Projektthema

- Veränderung der Schadstoffkonzentration abhängig von Weg & Zeit
- Bsp. Fabrik zu Wohnsiedlung

Projekthalt

- 2 „Gruppen“
- Theoretische Gruppe
 - Rechnen
- Praktische Gruppe
 - Anwenden
 - Programmieren
 - Visualisieren

Theoretische Gruppe

- Ohne Diffusion (κ):

- $c(x_0, t_0 + \Delta t) = -U \frac{c(x_0 + \Delta x, t_0) - c(x_0, t_0)}{\Delta x} \Delta t + c(x_0, t_0)$

- Mit Diffusion (κ):

- $c(x_0, t_0 + \Delta t) = \frac{\kappa}{2} * \frac{c(x_0 + \Delta x, t_0) - c(x_0, t_0) - c(x_0, t_0) + c(x_0 + \Delta x, t_0)}{\Delta x^2} - U \frac{c(\dots)}{\Delta x} \dots$

- Mit variablem Wind:

- $c(t_0 + \Delta t, x_0) = -\Delta t * \frac{U(x_0 + \Delta x) - U(x_0 - \Delta x)}{2\Delta x} c(t_0, x_0) - \dots$

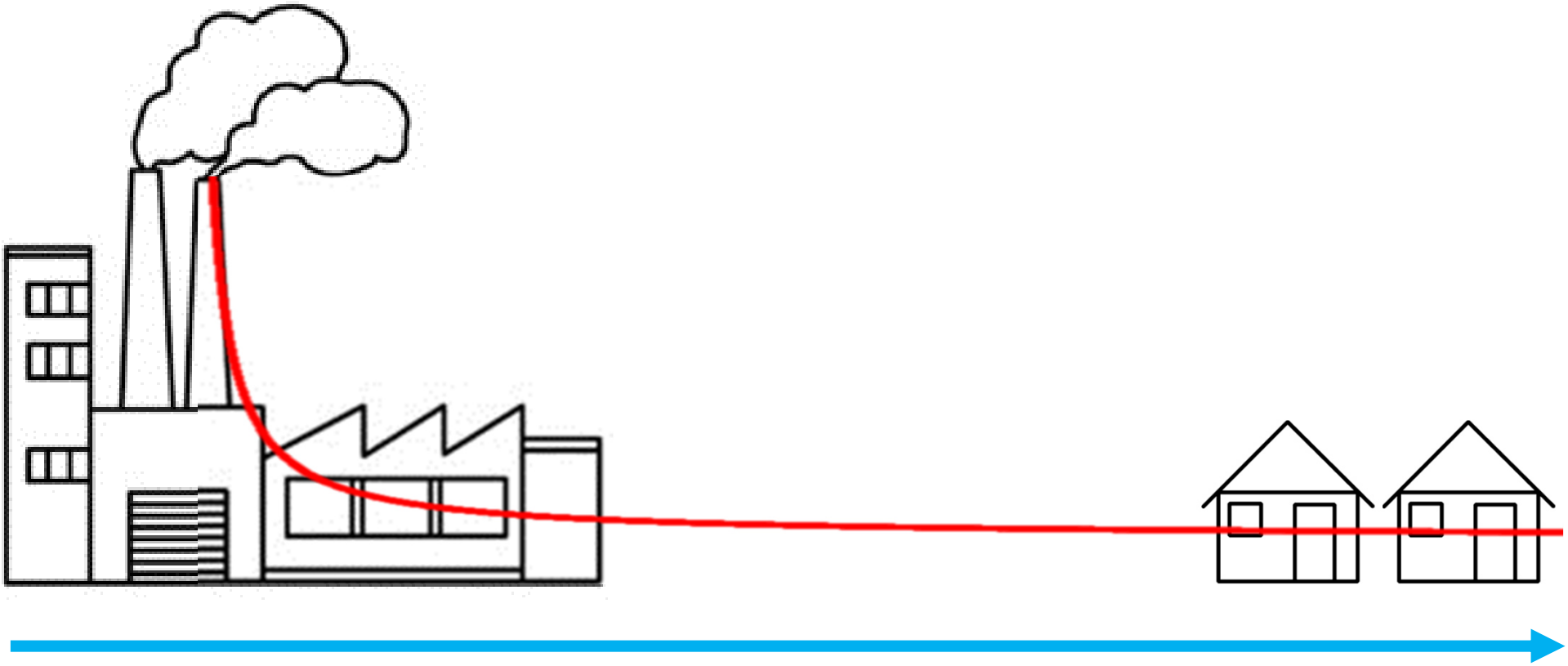
Bedingungen

- 1. Bedingung nach „Hofer & Hager“

- $0 \leq U \frac{\Delta t}{\Delta x} \leq 1$

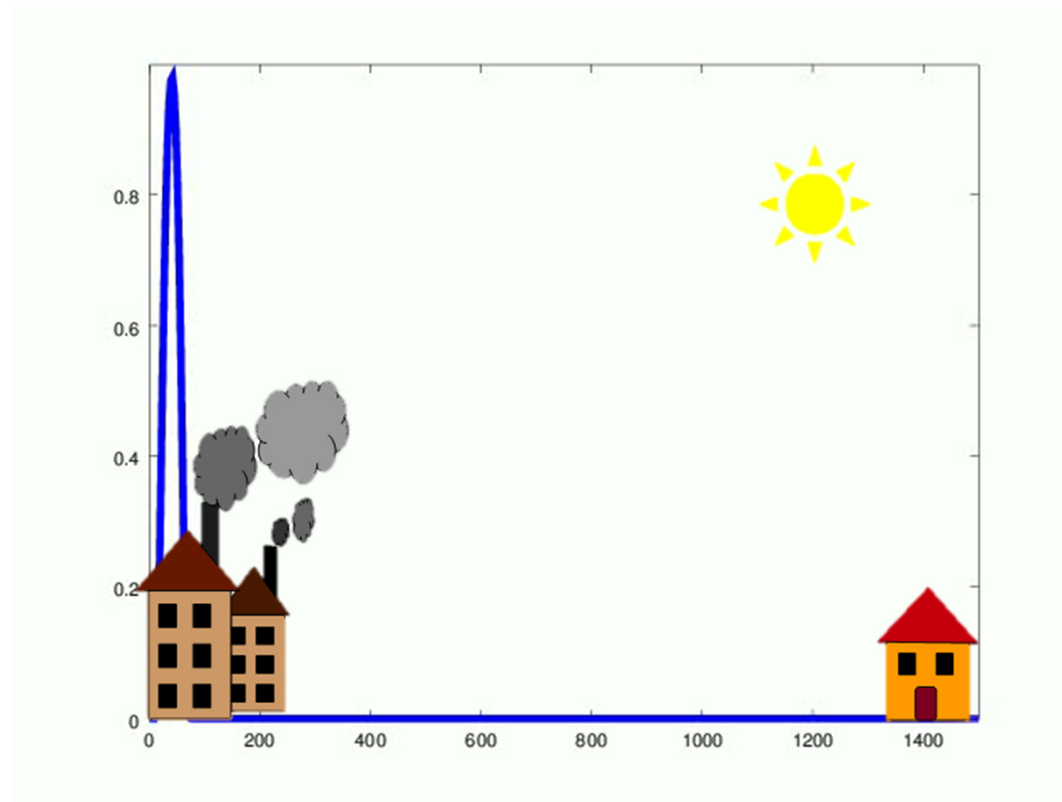
- 2. Bedingung nach „Hofer & Hager“

- $2 \frac{\Delta t}{\Delta x^2} (\kappa + \Delta x * U) \leq 1$

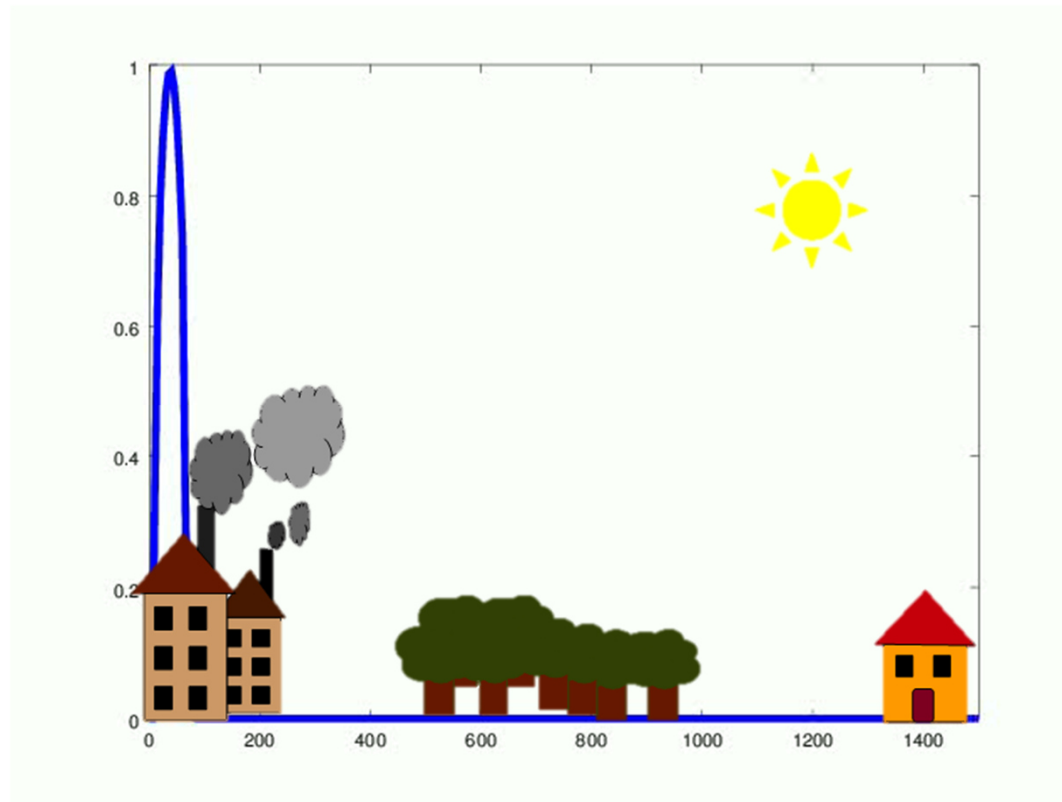


Wind

2D-Animation



2D-Animation mit variablem Wind



Fehler

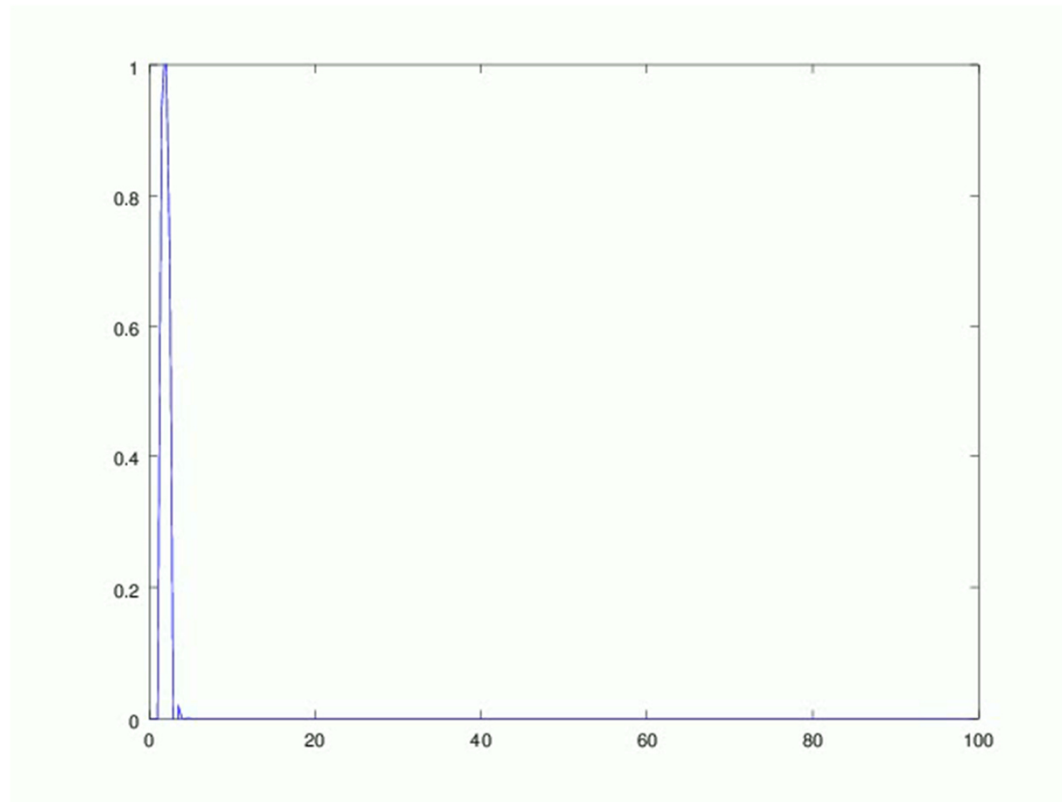
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error: C:/Users/Franziska/Documents/hlbla St. Florian/Schadstoffe/r³ckwörtsDQ.m: subscript indices must be either positive integers less than 2^31 or logicals
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error: called from
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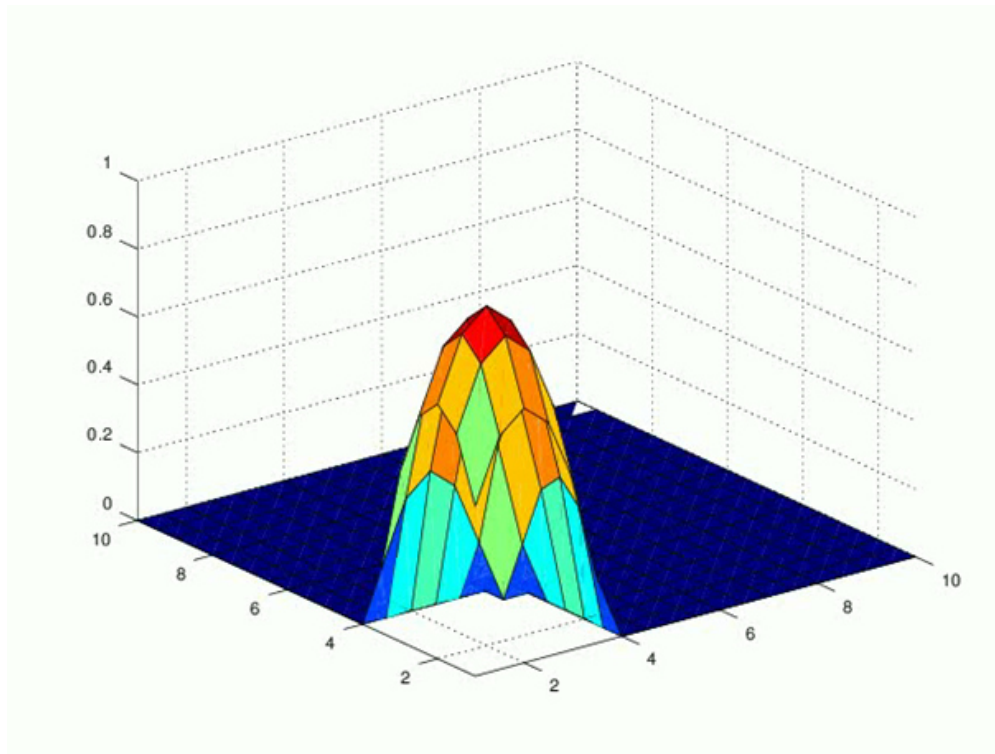
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    C:/Users/Franziska/Documents/hlbla St. Florian/Schadstoffe/r³ckwörtsDQ.m at line 15 column 9
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>> |
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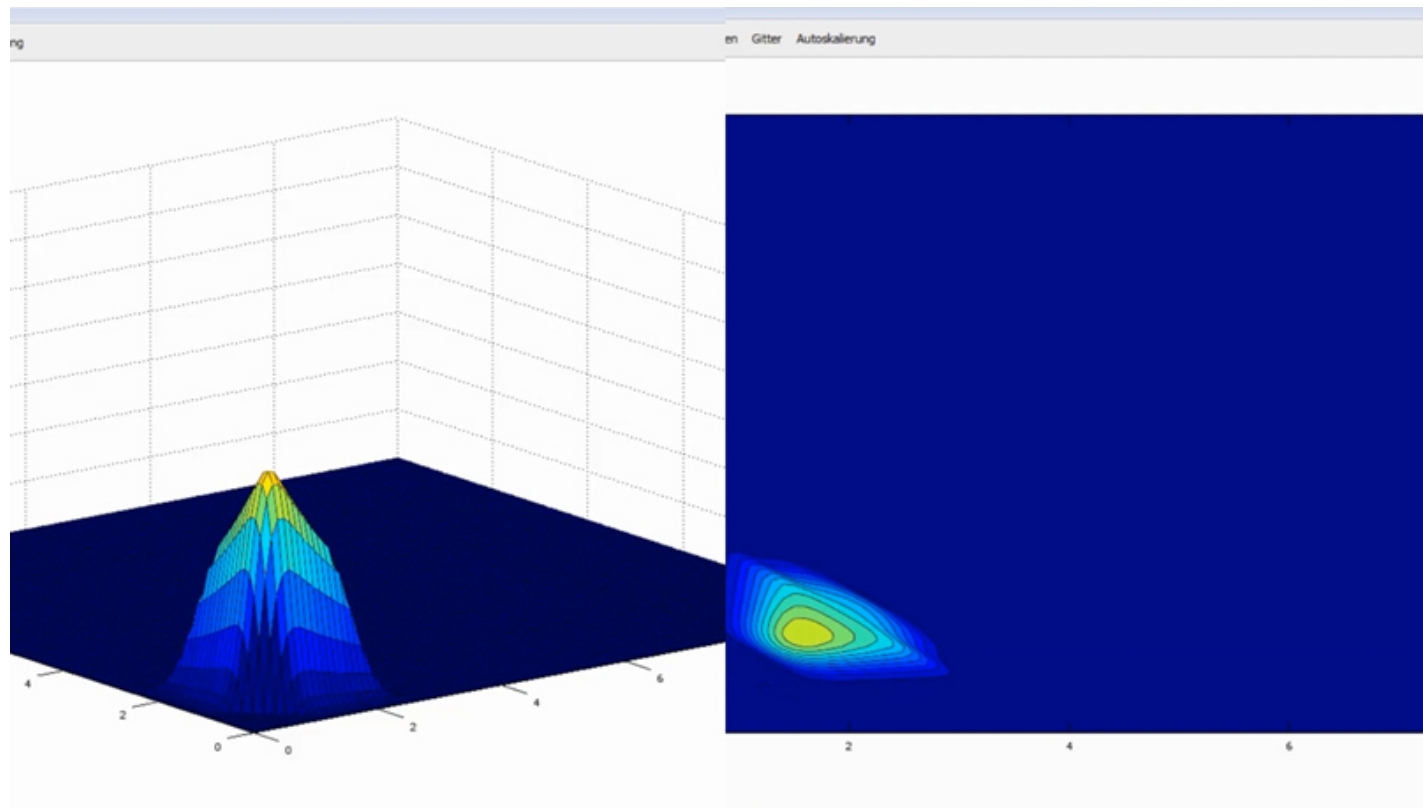
Fehler



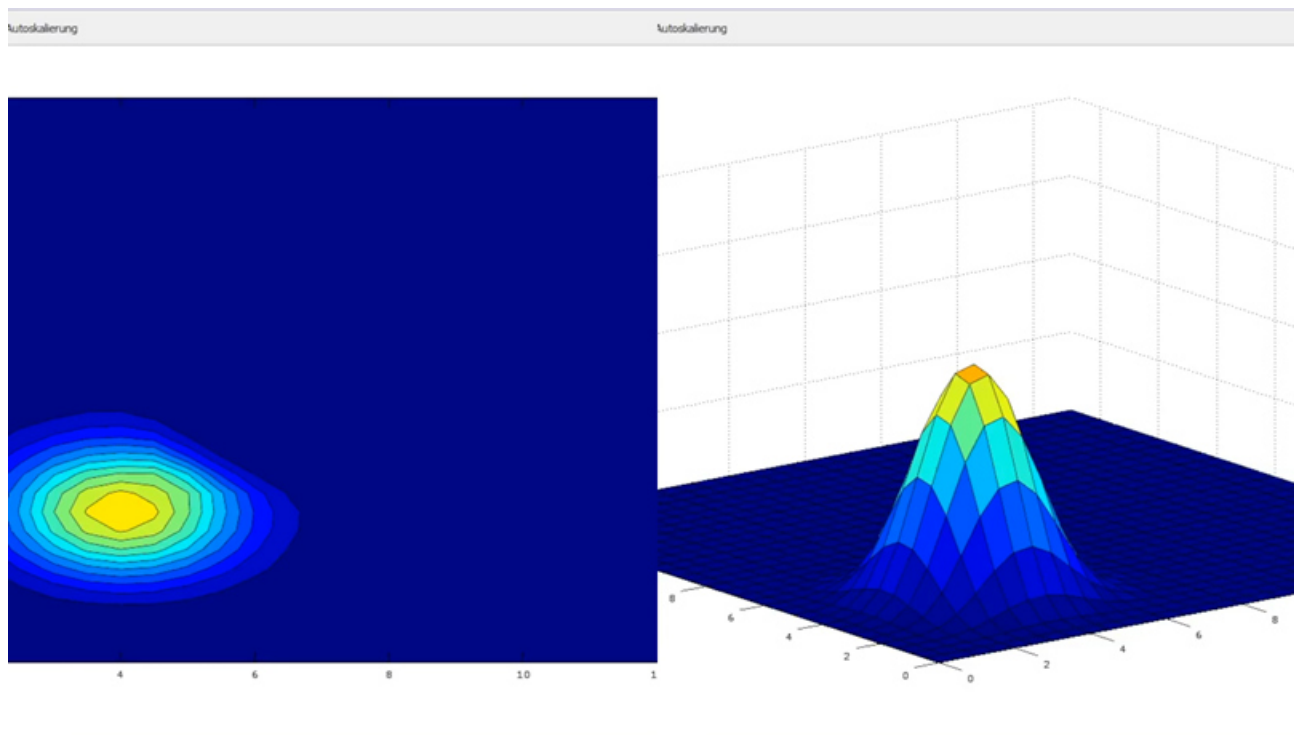
3D-Animation



3D-Animation



3D-Animation mit variablem Wind



Notizen

Projektwoche Angewandte Mathematik 2016

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$\frac{\partial^2 f}{\partial x^2}(x_0) = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

$$\frac{\partial c}{\partial t}(x, t) + U \cdot \frac{\partial c}{\partial x}(x, t) = 0$$

$$\frac{\partial c}{\partial t}(x_0, t_0) = \lim_{\Delta x \rightarrow 0} \frac{f\left(\frac{x_0}{\Delta x} + \Delta x, \frac{x_0}{\Delta x}\right) - f\left(\frac{x_0}{\Delta x}, \frac{x_0}{\Delta x}\right)}{\Delta x}$$

Vorwärts: $\frac{\partial c}{\partial t} \approx \frac{c(t_{i+1}) - c(t_i)}{t_{i+1} - t_i}$

Vorwärts: $\frac{\partial c}{\partial t} \approx \frac{c(t_{i+1}, x_{i+1}) - c(t_i, x_{i+1})}{t_{i+1} - t_i}$

t_0, t_1, t_2
 $0, 1, 2$

U: $\frac{\partial c}{\partial x} \approx \left(\frac{c(x_{i+1}, t_{i+1}) - c(x_i, t_{i+1})}{x_{i+1} - x_i} \right) \cdot U$

$$\frac{c(t_{i+1}, x_i) - c(t_i, x_i)}{t_{i+1} - t_i} + U \cdot \left(\frac{c(x_{i+1}, t_i) - c(x_i, t_i)}{x_{i+1} - x_i} \right) = 0$$

$$t_0 \quad \frac{c(t_1, x_i) - c(t_0, x_i)}{t_1 - t_0} + U \cdot \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{x_{i+1} - x_i} \right) = 0$$

$$\frac{c(t_1, x_i) - c(t_0, x_i)}{\Delta t} = -U \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{\Delta x} \right)$$

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$$c(t_1, x_i) = -\Delta t \cdot U \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{\Delta x} \right) + c(t_0, x_i)$$

Notizen...

③

$$y^{(n)} = e^{ax} = y^1 e^{ax} = U \cdot \frac{d^{n-1}}{dx^{n-1}} (e^{ax}) = U \cdot (a^{n-1} e^{ax}) = a^{n-1} U e^{ax}$$

$y^{(n)} = a^n y$
 $a^n y = a^n U e^{ax}$
 $a^n U e^{ax} = a^n U e^{ax}$
 $U = \frac{1}{a^n}$

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$c(x_1, t_1) = k \cdot \Delta t \cdot c(x_1, t_1) + \frac{1}{2} a \cdot (\Delta t)^2 \cdot c(x_1, t_1) + \dots$
 $\frac{1}{2} a \cdot (\Delta t)^2 \cdot c(x_1, t_1) = \frac{1}{2} a \cdot (\Delta t)^2 \cdot c(x_1, t_1)$
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Aufgabe A.11
 $I \int c(t, x) = -a t \cdot U \left(\frac{c(x_1, t_1) - c(x_2, t_1)}{a} \right) + c(t, x_1)$
 $t \in [0, T]$
 $x \in [0, 10]$
 $U = 1$
 $c_1(x) = \begin{cases} (x-1) \cdot (x-3) = x^2 - 3x + 3 & \text{if } x \in [1, 3] \\ 0 & \text{else} \end{cases}$
 $a = 1$
 $\Delta x = 0.5$
 $I \int c(t, x) = -a t \cdot U \left(\frac{c(x_1, t_1) - c(x_2, t_1)}{a} \right) + c(t, x_1)$
 $\frac{\partial c}{\partial t}(x, t) = K \cdot \frac{\partial^2 c}{\partial x^2}(x, t) + U \cdot \frac{\partial c}{\partial x}(x, t) = 0$
 $K \cdot \frac{\partial^2 c}{\partial x^2}(x, t) + U \cdot \frac{\partial c}{\partial x}(x, t) = 0$

$\frac{\partial^2 c}{\partial x^2} \approx \frac{\partial^2 c}{\partial x^2}(x_{i+1}, t_i) - \frac{\partial^2 c}{\partial x^2}(x_i, t_i)$
 $\approx \frac{c(x_{i+2}, t_i) - c(x_i, t_i)}{2 \Delta x} - \frac{c(x_{i+1}, t_i) - c(x_{i-1}, t_i)}{2 \Delta x}$
 $\approx \frac{c(x_{i+2}, t_i) - 2c(x_i, t_i) + c(x_{i-2}, t_i)}{2 \Delta x^2}$
 $\frac{\partial c}{\partial t}(x, t) = K \cdot \frac{\partial^2 c}{\partial x^2}(x, t) + U \cdot \frac{\partial c}{\partial x}(x, t) = 0$
 $\frac{c(x_i) - c(x_{i-1}))}{\Delta x} = K \cdot \frac{c(x_{i+2}, t_i) - 2c(x_i, t_i) - c(x_{i-2}, t_i)}{2 \Delta x^2} + U \cdot \frac{c(x_{i+1}, t_i) - c(x_{i-1}, t_i)}{\Delta x} = 0$
 $\frac{c(x_{i+2}, t_i) - c(x_i, t_i)}{\Delta x} = K \cdot \frac{c(x_{i+2}, t_i) - 2c(x_i, t_i) - c(x_{i-2}, t_i)}{2 \Delta x^2} + U \cdot \frac{c(x_{i+1}, t_i) - c(x_{i-1}, t_i)}{\Delta x}$

$(x-10) \cdot (x-6) = x^2 - 16x + 60$
 $x^2 - 70x + 600$
 $\frac{1}{\Delta x} \cdot \Delta x$
 $a \cdot \frac{dx}{dt} = 15 \cdot \frac{c - 2c + a}{\Delta x} + 7 \cdot \frac{b - a}{\Delta x} = 0$
 $c \cdot \frac{dx}{dt} = 15 \cdot \frac{2c + b}{\Delta x} + 7 \cdot \frac{c - b}{\Delta x} = 0$
 $\frac{dx}{dt} \cdot (b-x) = 15c - 3b + 15a + 7(2c-b) = 0$
 $\frac{dx}{dt} \cdot x \cdot dx = 10 \cdot 15 \cdot dx + 15 \cdot dx + 7 \cdot dx = 0$
 $\frac{dx}{dt} \cdot (15 \cdot dx - 2 \cdot dx) = b \cdot (dx - 3 \cdot dx) + 7 \cdot dx = 0$
 $\frac{dx}{dt} \cdot (c-x) = 15 \cdot \frac{b-c}{\Delta x} + 7 \cdot \frac{c-b}{\Delta x} = 0$
 $\frac{dx}{dt} \cdot x \cdot dx = 15 \cdot dx + 7 \cdot dx = 0$
 $c \cdot \frac{dx}{dt} \cdot (15 \cdot dx - 2 \cdot dx) = c \cdot (dx - 3 \cdot dx) + 7 \cdot dx = 0$

NOTIZEN!!!

$$\frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} = \frac{c(x_1, t) - c(x_2, t)}{\Delta x} \cdot \frac{1}{2}$$

$$\frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} - \frac{c(x_2, t) - c(x_1, t)}{2 \Delta x} = k$$

$$\left(\frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} - \frac{c(x_2, t) - c(x_1, t)}{2 \Delta x} \right) \cdot \frac{1}{2} = k$$

$$\frac{c(x_1, t) - c(x_2, t)}{2 \Delta x}$$

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$$\frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} = \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x}$$

$$\frac{d \cdot x_1}{dt} = a \cdot x_1$$

$$\frac{d \cdot x_2}{dt} = 1.5 \cdot k \cdot 2a \cdot x_2 = 2 \cdot \frac{a \cdot k \cdot x_2}{dt} = 0$$

$$\frac{d \cdot x_3}{dt} = 1.5 \cdot k \cdot 3a + 1.5 \cdot k \cdot 2a \cdot x_3 = 2a \cdot 2 \cdot k \cdot x_3 = 0$$

$$\frac{d(x_1 - x_2)}{dt} = 1.5 \cdot k \cdot 2a \cdot x_2 - 2 \cdot \frac{a \cdot k \cdot x_2}{dt} = 0$$

$$\frac{d(x_2 - x_3)}{dt} = 1.5 \cdot k \cdot 3a + 1.5 \cdot k \cdot 2a \cdot x_3 - 2a \cdot 2 \cdot k \cdot x_3 = 0$$

$$2 \cdot \frac{d(x_1 - x_2)}{dt} = 1.5 \cdot k \cdot 2a \cdot x_2 - 2 \cdot \frac{d(x_2 - x_3)}{dt} = 0$$

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$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = -U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x}$$

$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = -U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} \cdot (t+\Delta t) + c(x_1, t)$$

$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = U \cdot c(x_1, t)$$

$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} = 0$$

$$c(x_1, t+\Delta t) = U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x} \cdot \Delta t + c(x_1, t)$$

$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x}$$

$$(x-2)^2 + (y-3)^2 = 2^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 4$$

$$x^2 - 4x + y^2 - 6y + 9 = 4$$

$$x^2 - 4x + y^2 - 6y + 9 = 4$$

$$\frac{\partial c}{\partial t} - k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) + u \frac{\partial c}{\partial x} = 0$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = 0$$

$$\frac{\partial c}{\partial t}(x, y) - k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = 0$$

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$$\frac{\partial c}{\partial t}(x, y) + u \frac{\partial c}{\partial x}(x, y) - k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) = 0$$

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$$\frac{c(x_1, t+\Delta t) - c(x_1, t)}{\Delta t} = -U \cdot \frac{c(x_1, t) - c(x_2, t)}{2 \Delta x}$$

$$k \frac{\partial^2 c}{\partial x^2}(x, y) = \frac{\partial}{\partial x} \left(\frac{\partial c}{\partial x} \right) \cdot (x, y)$$

$$\frac{\partial c}{\partial t}(x, y) + u \frac{\partial c}{\partial x}(x, y) - k \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) = 0$$

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