Numerische Mathematik

Mathematik und Umwelt: Schadstoffsimulationen

Projektthema

- Veränderung der Schadstoffkonzentration abhängig von Weg & Zeit
- Bsp. Fabrik zu Wohnsiedlung

Projektinhalt

- 2 „Gruppen“
- Theoretische Gruppe
  - Rechnen
- Praktische Gruppe
  - Anwenden
  - Programmieren
  - Visualisieren

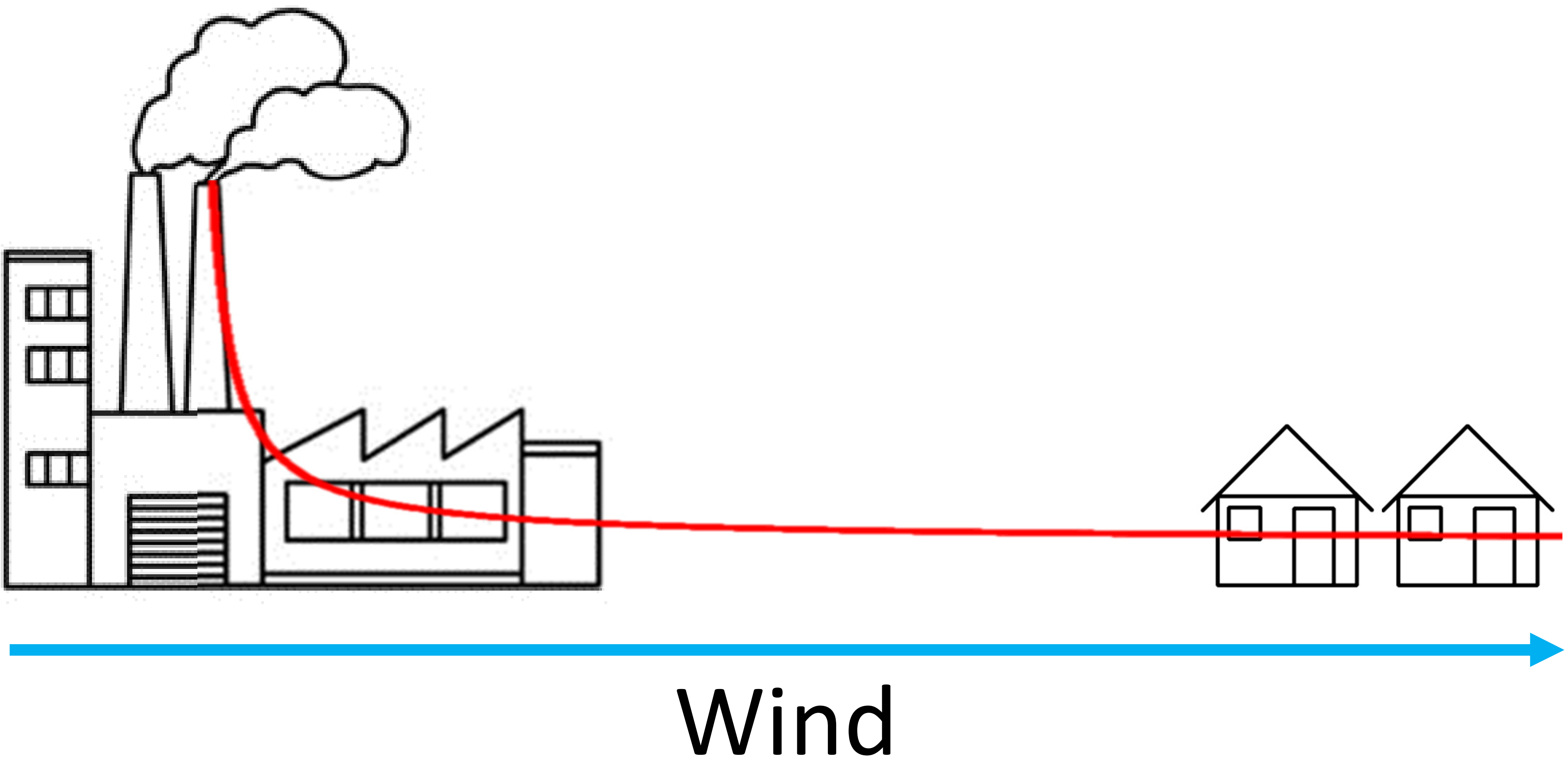
Theoretische Gruppe

- Ohne Diffusion (κ):
- $c(x_0, t_0 + \Delta t\ ) = -U\,\frac{c(x_0+\Delta x, t_0)-c(x_0, t_0)}{\Delta x}\,\Delta t + c(x_0, t_0)$
- Mit Diffusion (κ):
- $c(x_0, t_0 + \Delta t) = \frac{\kappa}{2} * \frac{c(x_0+\Delta x, t_0)-c(x_0, t_0)-c(x_0, t_0)+c(x_0+\Delta x, t_0)}{\Delta x^2} - U\,\frac{c(\ldots}{\Delta x}\ldots$
- Mit variablem Wind:
- $c(t_0 + \Delta t, x_0) = -\Delta t * \frac{U(x_0+\Delta x)-U(x_0-\Delta x)}{2\Delta x}\,\mathrm{c}(t_0, x_0) - \ldots$

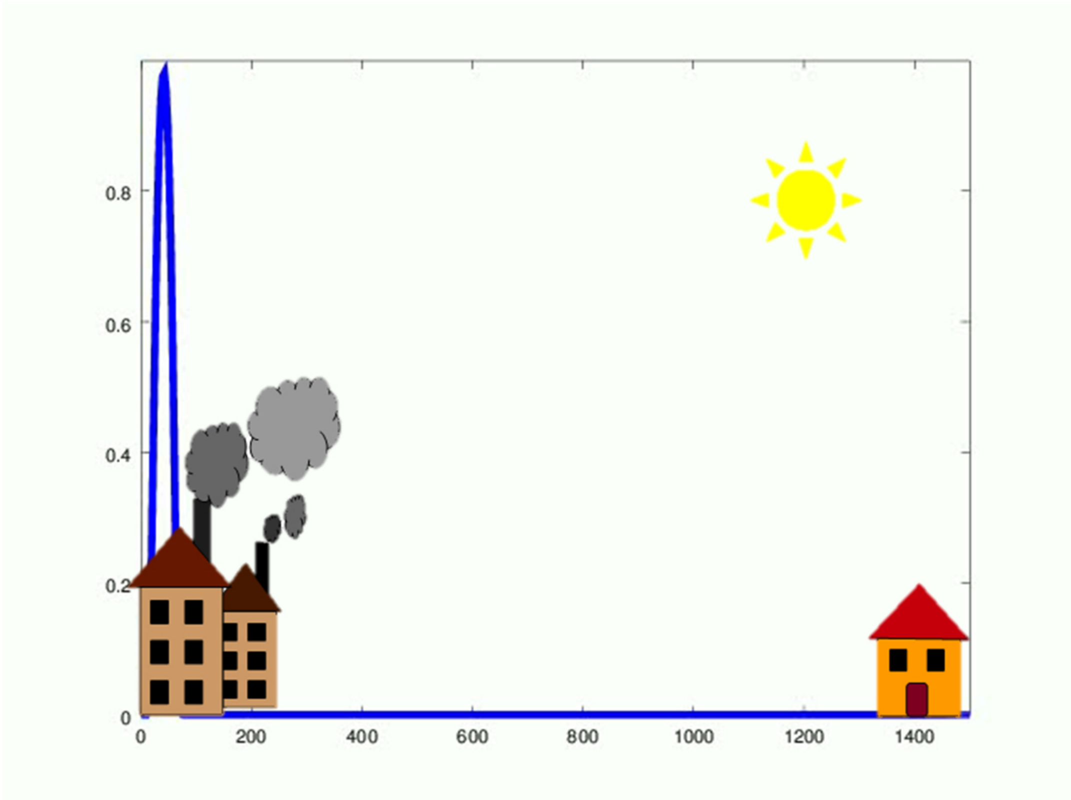
Bedingungen

- 1. Bedingung nach „Hofer & Hager“
- $0 \leq U \frac{\Delta t}{\Delta x} \leq 1$

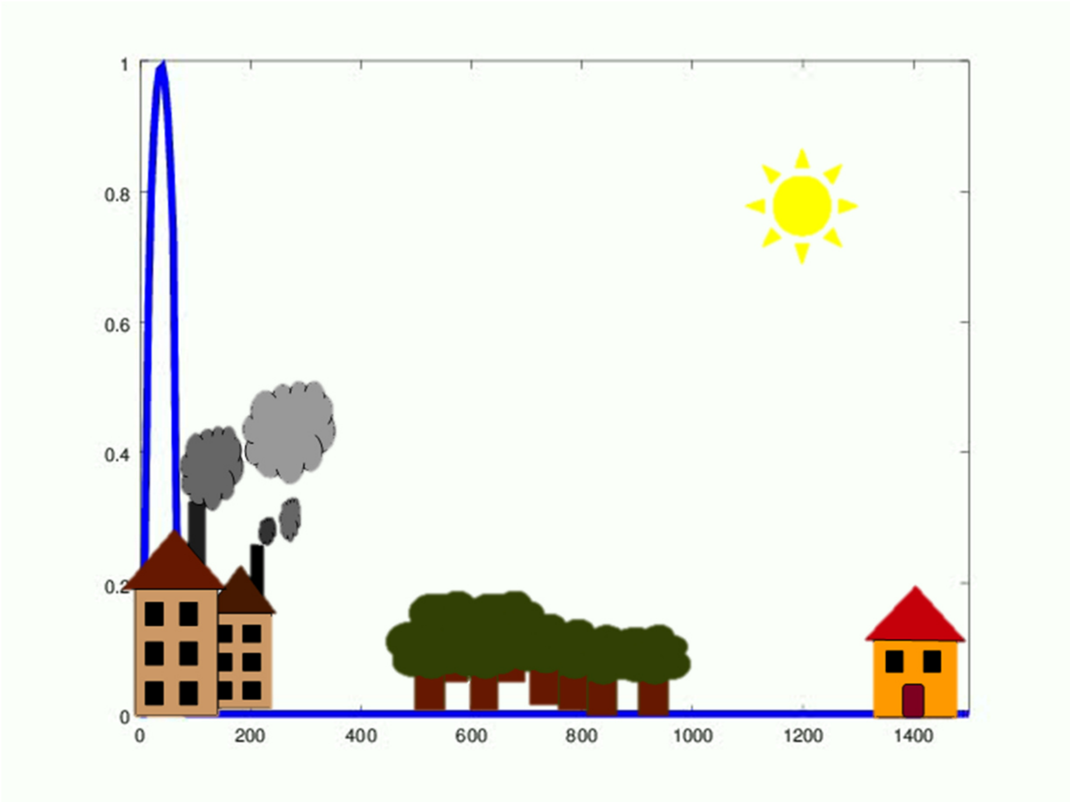
- 2. Bedingung nach „Hofer & Hager“
- $2 \frac{\Delta t}{\Delta x^2} (\kappa + \Delta x * U) \leq 1$


Wind

2D-Animation



2D-Animation mit variablem Wind



Fehler

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gers less than 2^31 or logicals
error: called from
    C:/Users/Franziska/Documents/hlbla St. Florian/Schadstoffe/r³ckwőrtsDQ.m at line 15 column 9

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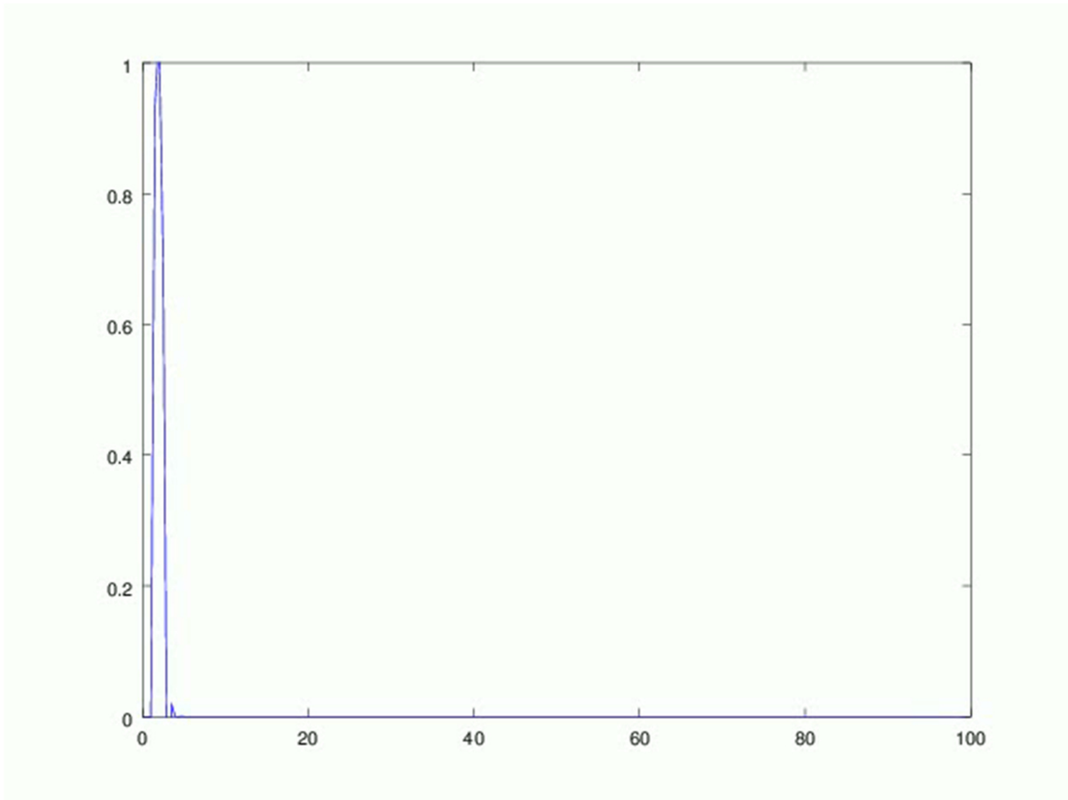
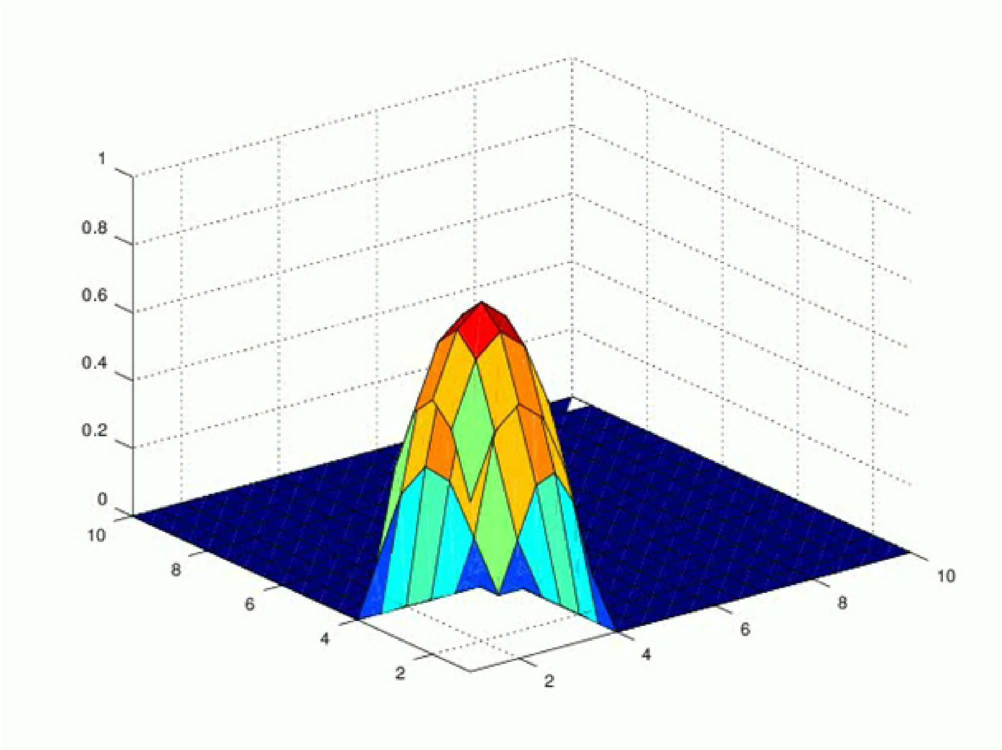
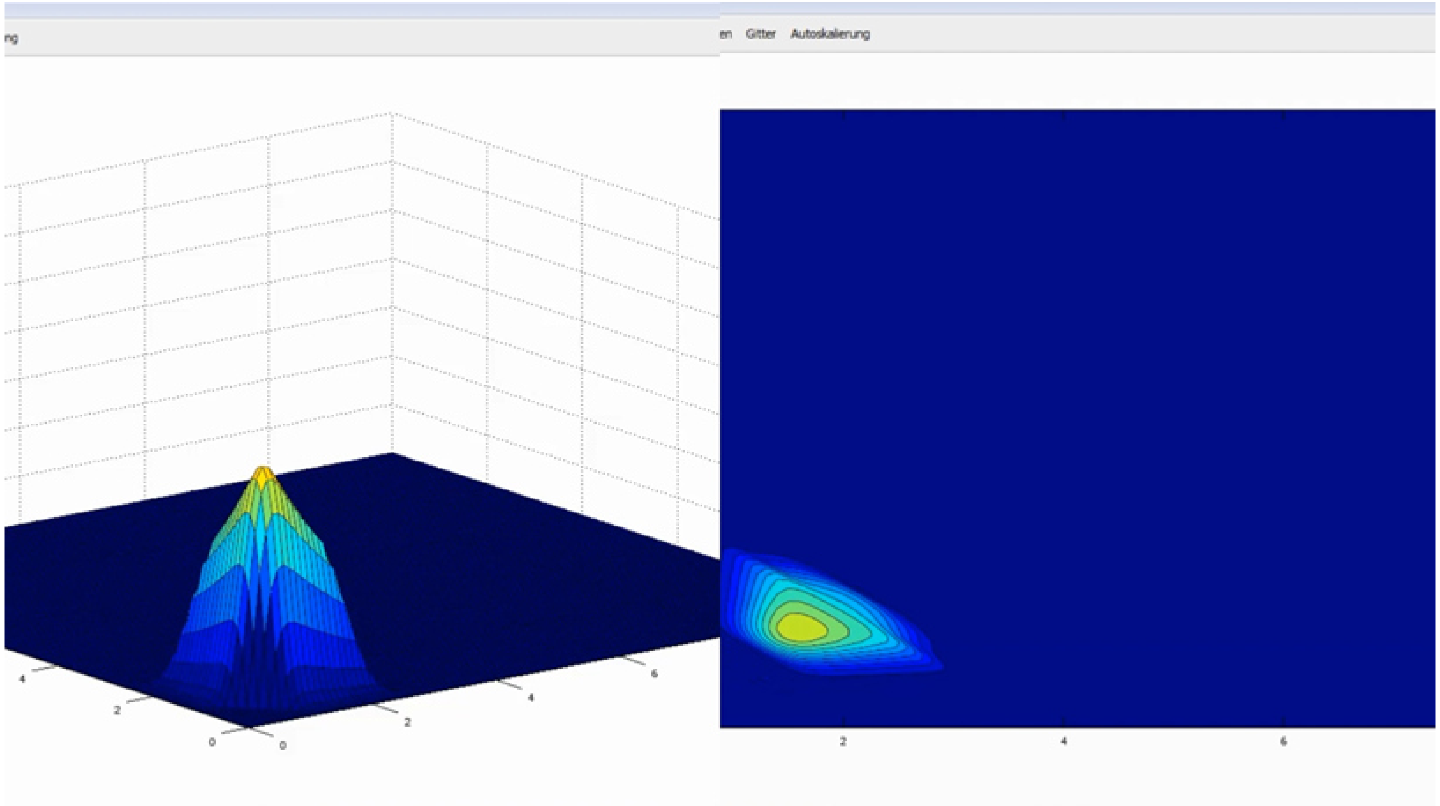
Fehler



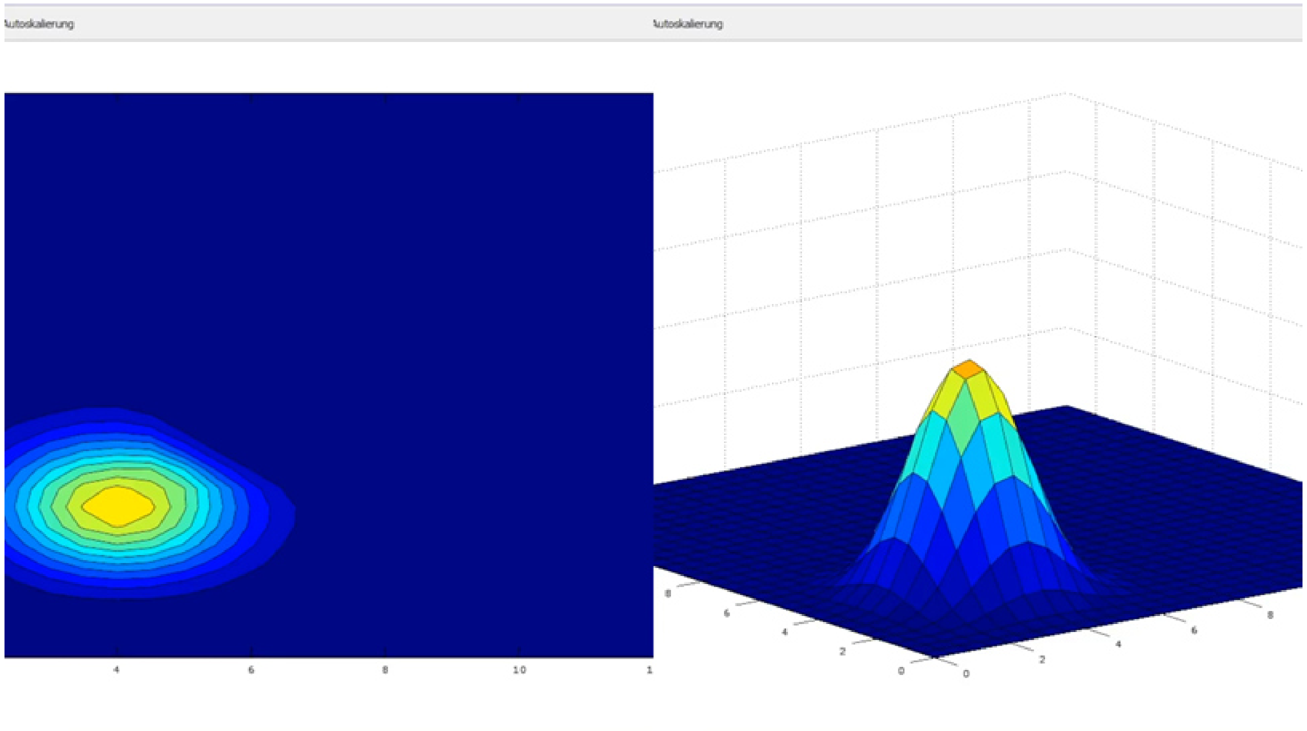
3D-Animation



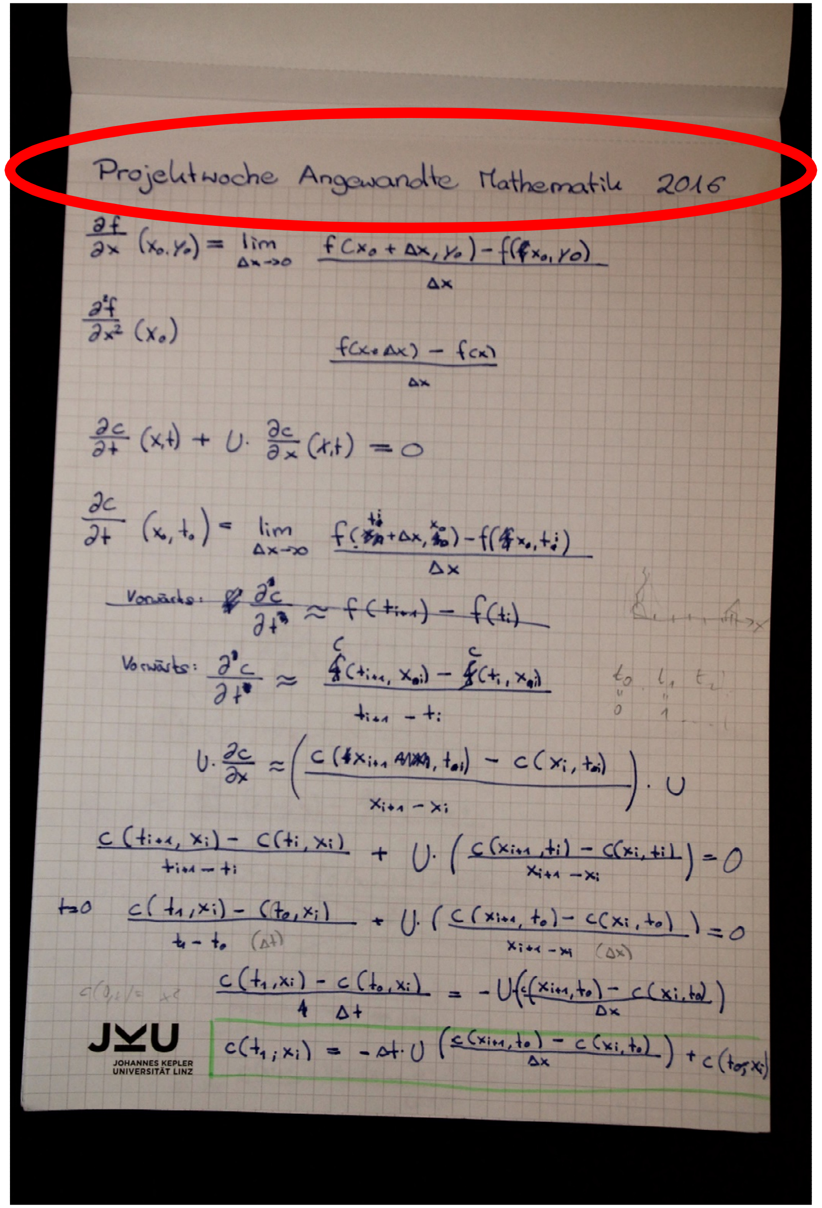
3D-Animation



3D-Animation mit variablem Wind



Notizen



Projektwoche Angewandte Mathematik 2016

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$\frac{\partial^2 f}{\partial x^2}(x_0)$$

$$\frac{f(x_0 + \Delta x) - f(x)}{\Delta x}$$

$$\frac{\partial c}{\partial t}(x,t) + U \cdot \frac{\partial c}{\partial x}(x,t) = 0$$

$$\frac{\partial c}{\partial t}(x_0, t_0) = \lim_{\Delta x \to 0} \frac{f(t_i + \Delta x, x_0) - f(x_0, t_i)}{\Delta x}$$

~~Vorwärts: $\frac{\partial c}{\partial t} \approx f(t_{i+1}) - f(t_i)$~~

Vorwärts: $\frac{\partial c}{\partial t} \approx \frac{c(t_{i+1}, x_{0i}) - c(t_i, x_{0i})}{t_{i+1} - t_i}$

$t_0 \quad t_1 \quad t_2$
$0 \quad 1$

$$U \cdot \frac{\partial c}{\partial x} \approx \left(\frac{c(x_{i+1}, t_{0i}) - c(x_i, t_{0i})}{x_{i+1} - x_i} \right) \cdot U$$

$$\frac{c(t_{i+1}, x_i) - c(t_i, x_i)}{t_{i+1} - t_i} + U \cdot \left(\frac{c(x_{i+1}, t_i) - c(x_i, t_i)}{x_{i+1} - x_i} \right) = 0$$

$t=0$: $$\frac{c(t_1, x_i) - c(t_0, x_i)}{t_1 - t_0 \ (\Delta t)} + U \cdot \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{x_{i+1} - x_i \ (\Delta x)} \right) = 0$$

$$\frac{c(t_1, x_i) - c(t_0, x_i)}{\Delta t} = -U \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{\Delta x} \right)$$

$$\boxed{c(t_1; x_i) = -\Delta t \cdot U \left(\frac{c(x_{i+1}, t_0) - c(x_i, t_0)}{\Delta x} \right) + c(t_0; x_i)}$$

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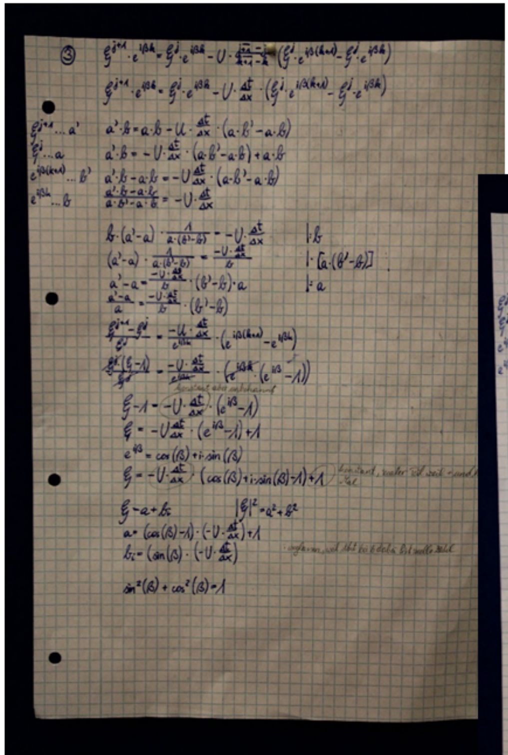
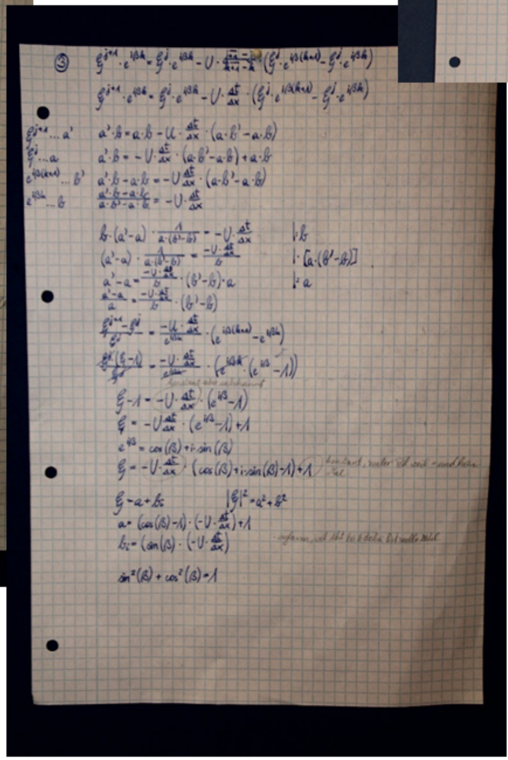
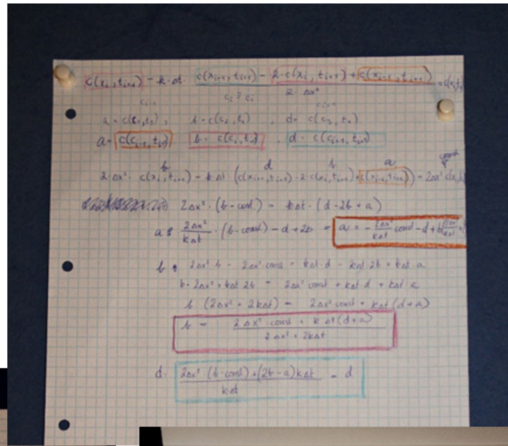
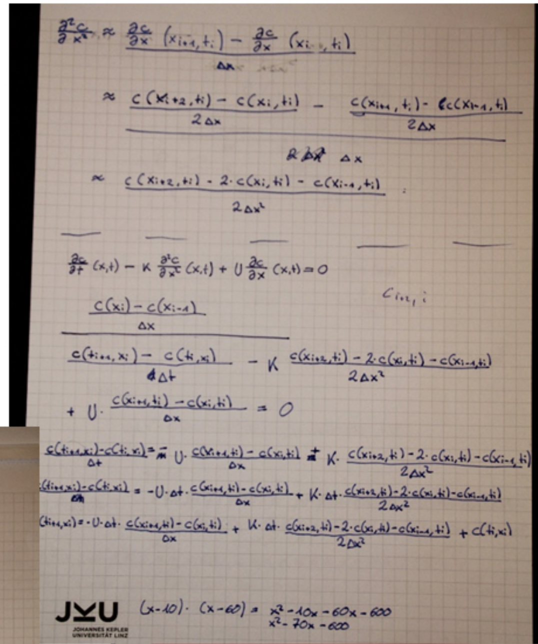
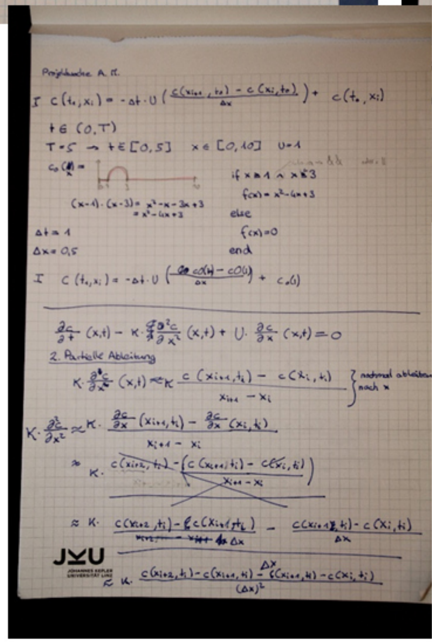
Notizen…

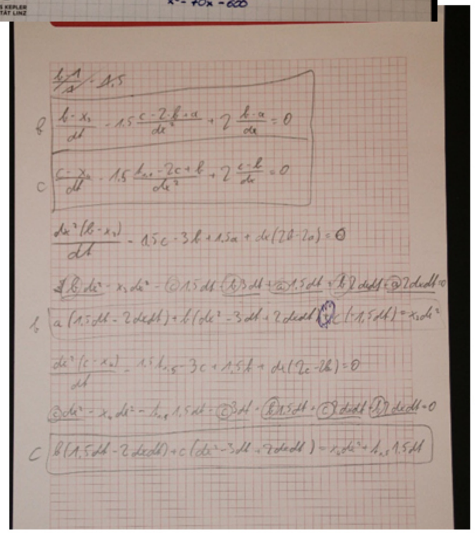












NOTIZEN!!!